Session Objectives

- Experience how proficiency in the tape diagram method can be developed in students and colleagues.
- Experience how to support understanding of various types of word problems as outlined in the Progressions document.
- Mathematical Practice 5: *Use appropriate tools strategically* and demonstrate this knowledge by using a tape diagram to solve problems.

Opening Exercise

- If you do not have tape diagram experience try to solve the problem on using a diagram or algebra.

94 children are in a reading club. One-third of the boys and three-sevenths of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?

Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Practice Set 1
- Practice Set 2
- Practice Set 3
- Practice Set 4
- Practice Set 5
- Practice Set 6

What is a Tape Diagram?

A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as strip diagrams, bar model, fraction strip, or length model.

(CCSSM Glossary, p. 87)

What is a Tape Diagram?

Grade 1: Math Drawings (1.OA.1, 1.OA.2)

Grade 2: Math Drawings (2.OA.1, 2.OA.2, 2.MD.5)

Grade 3: Visual Fraction Model (3.NF.3a-d)

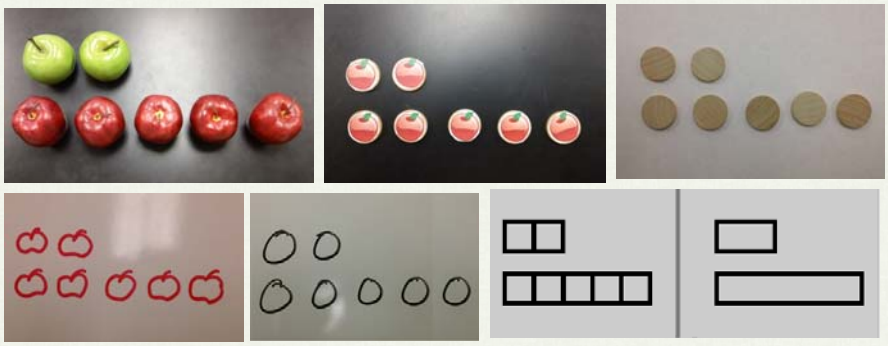
Grade 4: Visual Fraction Model (4.NF.3, 4.NF.4, 4.OA.2)

Grade 5: Visual Fraction Model (5.NF.2-4, 6, 7)

Grade 6: Tape Diagrams (6.RP-3) Visual Fraction Model (6.NS-1)

Grade 7: Visual Model for Problem Solving (7RP1-3)
Number Line Diagram (7.NS-1)

Developing Conceptual Understanding
Concrete → Pictorial → Visualization → Abstract



2 apples + 5 apples = 7 apples

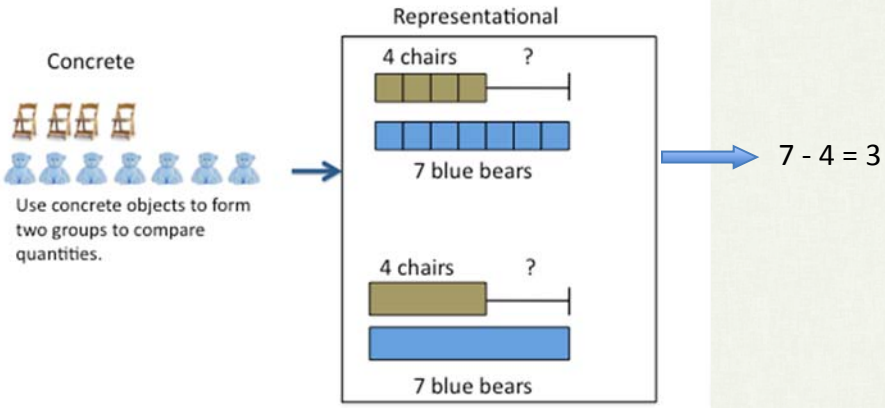
2 + 5 = 7

S = J - 3

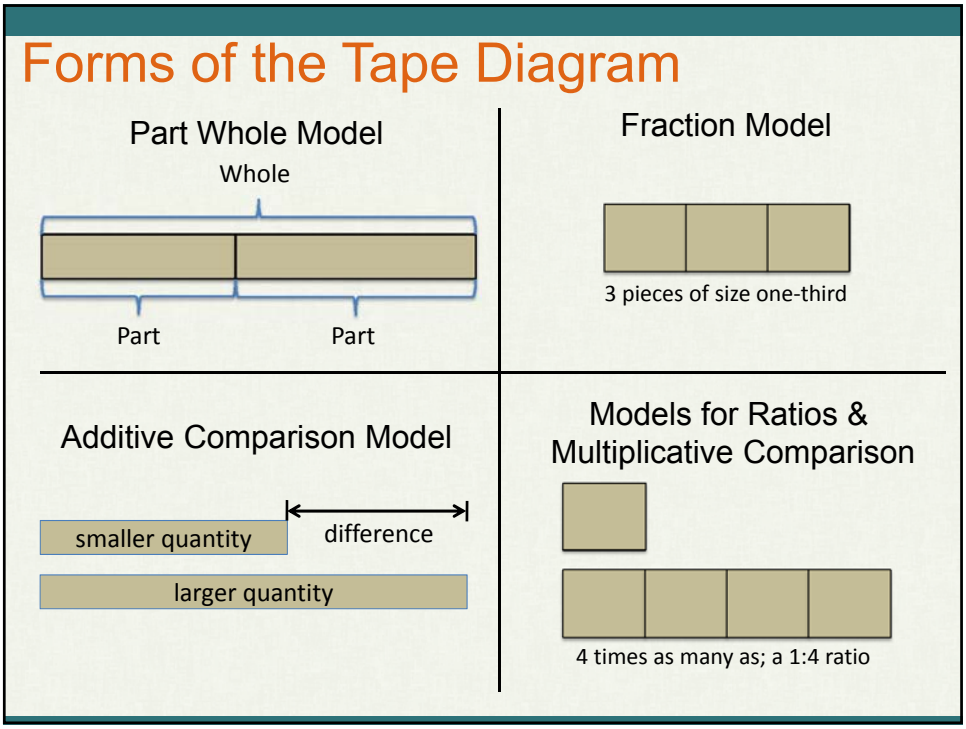
S + J = 7

Tape Diagrams: The Missing Link

Natalie has 7 blue bears and 3 chairs. How many more blue bears does Natalie have than chairs?



Forms of the Tape Diagram



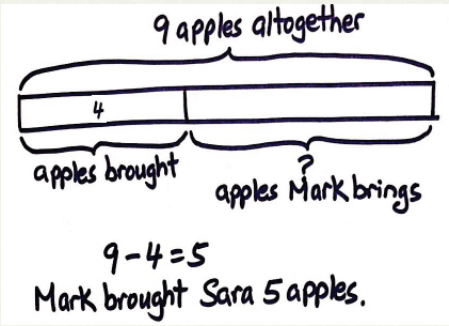
Using Tape Diagrams

- Promote **perseverance** in reasoning through problems.
- Develop students' independence in asking themselves:
  - "Can I draw something?"
  - "What can I label?"
  - "What do I see?"
  - "What can I learn from my drawing?"

Tape Diagrams

Early Examples

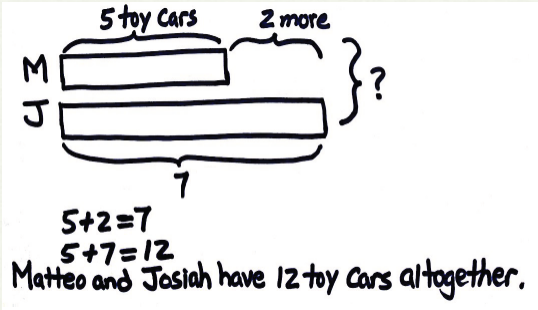
Sara brought 4 apples to school. After Mark brings her some more apples, she has 9 apples altogether. How many apples did Mark bring her?



Tape Diagrams

Early Examples

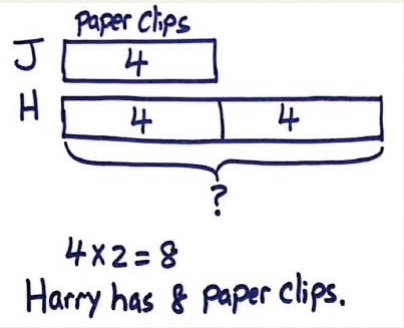
Matteo has 5 toy cars. Josiah has 2 more than Matteo. How many cars do Matteo and Josiah have altogether?



Tape Diagrams

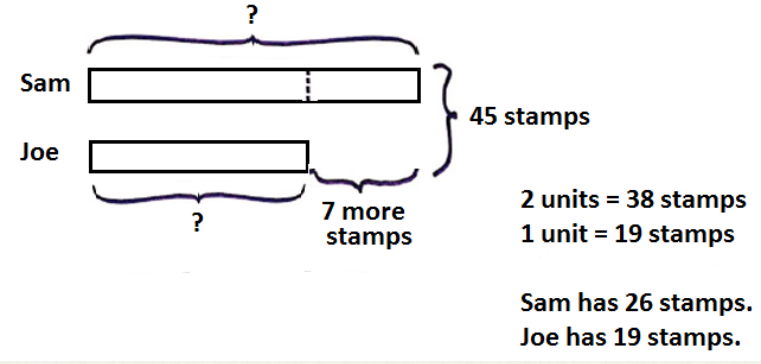
Early Examples

Example D: Jose has 4 paper clips. Harry has twice as many paper clips as Jose. How many paper clips does Harry have?



Example A:

Sam has 7 more stamps than Joe. They have 45 stamps altogether. How many stamps does each boy have?



Example A:

Rose has a vase with 13 flowers. She puts 7 more flowers in the vase. How many flowers are in the vase?

G1 M4 L19

Example B:

Sixteen strawberries are in a basket for Peter and Julio. Peter ate 8 of them. How many are there for Julio to eat?

G1 M4 L20

Example B:

Nine dogs were playing at the park. Some more dogs came to the park. Then there were 11 dogs. How many more dogs came to the park?

G1 M4 L20

Example C:

Ben and Peter caught 17 tadpoles. They gave some to Anton. They have 4 tadpoles left. How many tadpoles did they give to Anton?

G1 M4 L21

Example D:

Some yellow beads were on Tamra's bracelet. After she put 14 purple beads on the bracelet, there were 18 beads. How many yellow beads did Tamra's bracelet have at first?

G1 M4 L20

Example E:

Kiana found some shells at the beach. She gave 8 shells to her brother. Now she has 9 shells left. How many shells did Kiana find at the beach?

G1 M4 L21

Example G:

Thirteen children are on the roller coaster. Three adults are on the roller coaster. How many people are on the roller coaster?

G1 M4 L20

Example G:

Emi has some goldfish. Tamra has 14 Beta fish. Tamra and Emi have 19 fish in all. How many goldfish does Emi have?

G1 M4 L21

Example F:

The students were playing with 7 balls on the playground. They accidentally kicked some of the balls into a big puddle and now some are muddy! What is one way the balls might look?

GK M4 L18

Stop to Reflect:

What did you notice about the problems in Problem Set 1?

Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  - Addition and Subtraction Models – Part Whole
- Problem Set 2
- Problem Set 3
- Problem Set 4
- Problem Set 5
- Problem Set 6

Example A:

Rose wrote 8 letters. Nikii wrote 12 letters. How many more letters did Nikii write than Rose?

G1 M6 L1

Example B:

Lee collected 13 eggs from the hens in the barn. Ben collected 18 eggs from the hens in the barn. How many fewer eggs did Lee collect than Ben?

G1 M6 L1

Example B:

Ben played 9 songs on his banjo. Lee played 3 more songs than Ben. How many songs did Lee play?

G1 M6 L1

Example C:

Rose saw 14 monkeys at the zoo. She saw 5 fewer monkeys than foxes. How many foxes did Rose see?

G1 M6 L17

Example D:

Emi planted 12 flowers. Rose planted 3 fewer flowers than Emi. How many flowers did Rose plant?

G1 M6 L2

Example E:

Peter has 8 more green crayons than yellow crayons. Peter has 10 green crayons. How many yellow crayons does Peter have?

G1 M6 L23

Stop to Reflect:

What did you notice about the problems in Problem Set 2?

Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  Addition and Subtraction Models: Part Whole
- Problem Set 2: Addition Comparison Problems
- Problem Set 3
- Problem Set 4
- Problem Set 5
- Problem Set 6

Example A:

Angela has 790 grams of cement. She poured 250 grams of sand into the mixture. Find the total weight of the new mixture in kilograms and grams.

G3-G5 Context

Example 9:

Stephen sifted out 250 grams of sand from a mixture of sand and gravel weighing 1.04 kilograms. How much did the remaining mixture weigh?

G3-G5 Context

Example 10:

How many kilograms did a mixture of 250 grams of sand and 790 grams of gravel weigh? Express the weight in decimal form.

G3-G5 Context

Example J:

At 7:00 AM Hovik measured the water in a bucket to be $1\frac{7}{8}$ inches deep. By 7:00 PM he measure the depth at $5\frac{3}{4}$ inches. How many inches of water were collected between 7:00 AM and 7:00 PM?

G3-G5 Context

Example 11:

A bucket contained $5\frac{3}{4}$ inches of water. After two days evaporating in the sun, it contained $1\frac{7}{8}$ inches. How many inches of water evaporated in two days?

G3-G5 Context

Example 12:

After a storm which lasted from 9:00 AM to 1:00 PM, a bucket contained $5\frac{3}{4}$ inches of water. How many inches of rain might have fallen before 11:00 AM and how many inches might have fallen after 11:00 AM? Write at least three possibilities.

G3-G5 Context

Example 13:

Mr. Ramos sold $560,090 worth of stock. If the total value of his remaining stock portfolio was $314,511, find the total value of his portfolio at first.

G3-G5 Context

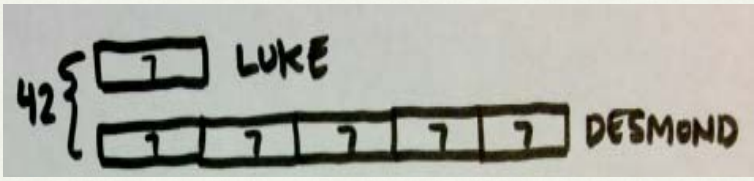
Example 14:

Mr. Ramos has $874,601 worth of stock in 2 accounts. He has $560,090 in one of the accounts. What is the value of the stock in the other account?

G3-G5 Context

Example L:

Desmond has 5 times as many toy cars as Luke. They have 42 cars altogether. How many cars does each boy have.



Example A:

A parking structure has 10 levels. There are 3 cars parked on each level. How many cars are parked in the structure?

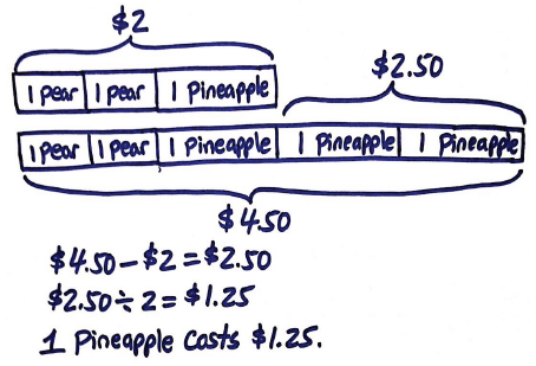
G3 M6 L18

Example B:

The total weight of a football and 10 tennis balls is 1 kg. If the weight of each tennis ball is 60 g, find the weight of the football.

Example M:

Two pears and a pineapple cost $2. Two pears and three pineapples cost $4.50. Find the cost of a pineapple.



Group Size Unknown

Example C:

Mark spends $16 on 2 video games. Each game costs the same amount. Find the cost of each game.

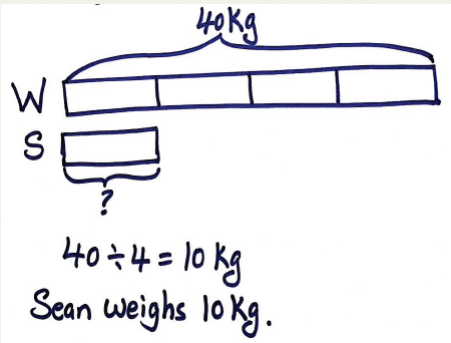
G3 M6 L12

Stop to Reflect:

What did you notice about the problems in Problem Set 3?

Example N:

William's weight is 40 kg. He is 4 times as heavy as his youngest brother Sean. What is Sean's weight?



Example 20:

Rosie had 17 yards of fabric. She used one-third of it to make a quilt. How many yards of fabric did Rosie use for the quilt?

G5 M4 L7

Example 21:

A calculator costs $12.75 and that is 5 times as much as a pair of socks. How much do the socks cost?

Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  Addition and Subtraction Models: Part Whole
- Problem Set 2: Addition Comparison Problems
- Problem Set 3:
  Multiplication and Division Models: Part Whole
- Problem Set 4
- Problem Set 5
- Problem Set 6

Example A:

There are 400 children at Park Elementary School. Park High School has 4 times as many students. How many students in all attend both schools?

G4 M3

Example B:

35 students ordered hamburgers. That is 5 times as many as the number of students who ordered a salad. How many students ordered a salad?

Example C:

Tiffany spent $\frac{1}{7}$ of her money on a teddy bear. If the teddy bear costs $28, how much money did she have at first?

Example D:

Sarah is 9 years old. Sarah's grandfather is 90 years old. Sarah's grandfather is how many times as old as Sarah?

G4 M1 L1

Stop to Reflect:

What did you notice about the problems in Problem Set 4?

Example ##:

Mr. Ramos sold some stock in the morning and $560,090 worth of stock from 11:00 AM to 2:00 PM. At 2:00 PM, he realized he had sold $874,601 worth of stock so far that day. What was the value of the stock he sold in the morning?

G3-G5 Context

Start Unknown

Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  - Addition and Subtraction Models: Part Whole
- Problem Set 2: Addition Comparison Problems
- Problem Set 3:
  - Multiplication and Division Models: Part Whole
- Problem Set 4: Multiplication Comparison Problems
- Problem Set 5
- Problem Set 6

Example A:

David spent 2/5 of his money on a storybook. The storybook cost $20. How much did he have at first?

Example B:

Max spent 3/5 of his money in a shop and 1/4 of the remainder in another shop. What fraction of his money was left? If he had $90 left, how much did he have at first?

Example C:

Alex bought some chairs. One third of them were red and one fourth of them were blue. The remaining chairs were yellow. What fraction of the chairs were yellow?

Example D:

Angela mixed $6\frac{7}{8}$ pounds of fertilizer into the soil of an avocado tree. She mixed one-fifth as much into the soil of a lemon tree. How much fertilizer did she mix into the soil of the lemon tree? Express your answer in pounds.

Example E:

Jim had 360 stamps. He sold 1/3 of them on Monday and 1/4 of the remainder on Tuesday. How many stamps did he sell on Tuesday?

Example D:

Three-fifths of Jan's money is twice as much as Lena's money. What fraction of Jan's money is Lena's money?

Example E:

Henry bought 280 blue and red paper cups. He used 1/3 of the blue ones and 1/2 of the red ones at a party. If he had an equal number of blue cups and red cups left, how many cups did he use altogether?

Example 24:

Angela mixed $1\frac{3}{8}$ pounds of fertilizer into the solid of a lemon tree. She mixed 5 times as much into the soil of an avocado tree. How many pounds of fertilizer did she mix into the soil of the avocado tree?

Larger Unknown

Example 25:

A pair of socks costs \$2.55. That is $\frac{1}{5}$ as much as a shirt, how much does the shirt cost?

Larger Unknown

Example 26:

54 oranges in the bag were perfect and 6 were rotten. What fraction as much were rotten as perfect?

Multiplier Unknown

Example 27:

When baking bread, each loaf calls for one-third cup of honey and 4 cups of flour. How many times as much flour is needed as honey?

Multiplier Unknown

Example 28:

When baking baklava, each tray calls for 3 cups of honey and 6 cups of walnuts. What fraction of the amount of flour is the honey?

Multiplier Unknown

Example XXXX:

A gift costs $1,272. Twenty-four students share the cost equally. How much does each person pay?

G3 M6 L12 Group Size Unknown

Stop to Reflect:

What did you notice about the problems in Problem Set 5?

Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  Addition and Subtraction Models: Part Whole
- Problem Set 2: Addition Comparison Problems
- Problem Set 3:
  Multiplication and Division Models: Part Whole
- Problem Set 4: Multiplication Comparison Problems
- Problem Set 5: Fraction Models: Part Whole
- Problem Set 6:

Example A:

The ratio of the length of Tom's rope to the length of Jan's rope was 3:1. The ratio of the length of Maxwell's rope to the length of Jan's rope was 4:1. If Tom, Maxwell and Jan have 80 feet of rope altogether, how many feet of rope does Tom have?

Example B:

The Business Direct Hotel caters to people who travel for different types of business trips. On Saturday night there is not a lot of business travel, so the ratio of the number of occupied rooms to the number of unoccupied rooms is 2:5. On Sunday night, the ratio changes to 6:1. If the Business Direct Hotel has 432 occupied rooms on Sunday night, how many unoccupied rooms does it have on Saturday night?

G6 M1 L6

Example B:

Lena finds two boxes of printer paper in the teacher supply room. The ratio of the packs of paper in Box A to the packs of paper in Box B is 4:3. If half of the paper in Box A is moved to Box B, what is the new ratio of packs of paper in Box A to Box B?

Example C:

Sana and Amy collect bottle caps. The ratio of the number of bottle caps Sana has to the number Amy has is 2 : 3. The ratio became 5 : 6 when Sana added 8 more bottle caps to her collection. How many bottle caps does Amy have?

Example D:

The ratio of songs on Jessa's phone to songs on Tessie's phone is 2 to 3. Tessie deletes half of her songs and now has 60 fewer songs than Jessa. How many songs does Jessa have?

Example F:

The ratio of the number of Ingrid's stamps to the number of Ray's stamps is 3 : 7. If Ingrid gives one-sixth of her stamps to Ray, what will be the new ratio of the number of Ingrid's stamps to the number of Ray's stamps?

Example E:

Jack and Matteo had an equal amount of money each. After Jack spent $38 and Matteo spent $32, the ratio of Jack's money to Matteo's money was 3 : 5. How much did each boy have at first?

Example F:

The ratio of the Gavin's money to Manuel's was 6 : 7. After Gavin spent two-thirds of his money and Manuel spent $39 Manuel had twice as much money as Gavin. How much money did Gavin have at first?

Example XXXX:

Mason and Laney ran laps to train for the long-distance running team. The ratio of the number of laps Mason ran to the number of laps Laney ran was 2 to 3. If Mason ran 4 miles, how far did Laney run?

If Laney ran 930 meters, how far did Mason run?

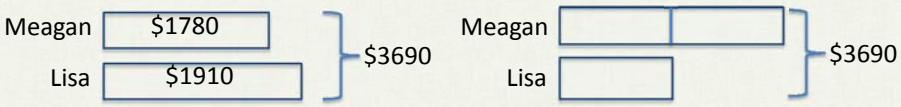
Example 31:

Josie took a long multiple-choice, end-of-year vocabulary test. The ratio of the number of problems she got incorrect to the number of problems she got correct is 2:9. If Josie missed 8 questions, how many did she get right?

G6 M1 L3

Example 34:

Meagan had $1780 and Lisa had $1910. Lisa gave some money to Meagan. In the end Meagan had twice as much money as Lisa. How much money did Lisa give to Meagan?



Example U:

Ingrid is mixing yellow and green paint together for a large art project. She uses a ratio of 2 pints of yellow paint for every 3 pints of green paint.

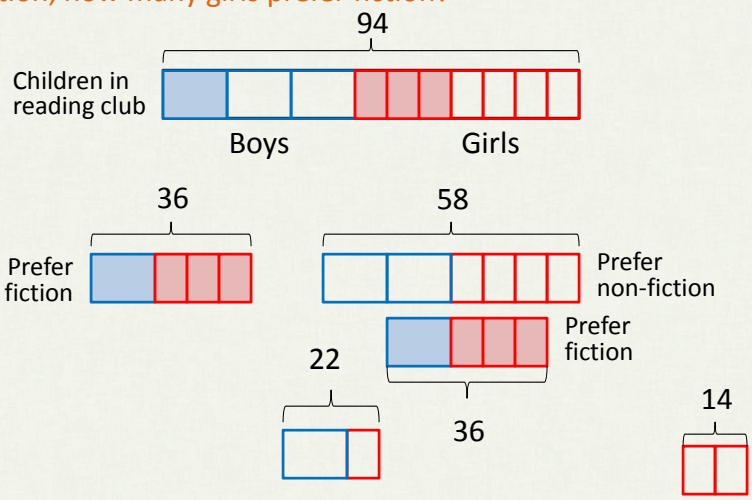
Stop to Reflect:

What did you notice about the problems in Problem Set 6?

Opening Exercise:

94 children are in a reading club. One-third of the boys and three-sevenths of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?

94 children are in a reading club. One-third of the boys and three-sevenths of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?



Alternative Opening Exercise

Jess spent one-third of her money on a cell phone, and two-fifths of the remainder on accessories. When she got home her parents gave her $350. The ratio of money she had in the end to the money she had before was 4:3. How much money did she have at first?

Agenda – Tape Diagrams

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- Problem Set 1:
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- Problem Set 3:
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- Problem Set 4: Multiplication Comparison Problems
- Problem Set 5: Fraction Models: Part Whole
- Problem Set 6: Ratio Models

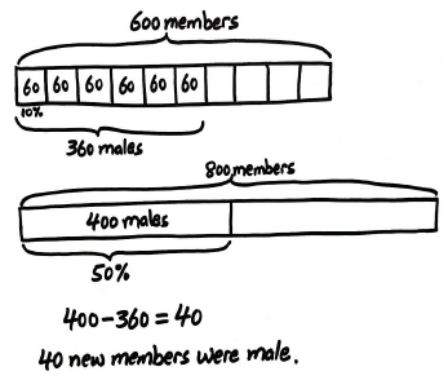
Double Number Line Diagrams

Example 1: Percentage Problems

Mia's weekly salary is $928. This is 80% of Sana's weekly salary. Find Sana's weekly salary.

Example 2:

A club had 600 members. 60% of them were males. When 200 new members joined the club, the percentage of male members was reduced to 50%. How many of the new members were males?



Double Number Line Diagrams

Example 2: Rate Problems

A photocopier can print 12 copies in 36 seconds. At this rate, how many copies can it print in 1 minute?

Double Number Line Diagrams

Example 3:

A school cafeteria has a restriction on the amount of sugary drinks available to students. Drinks may not have more than 25 grams of sugar. Based on this restriction, what is the largest size cola (in ounces) the cafeteria can offer to students?

Using Variables with Tape Diagrams

Example 4:

Mary had $460. She bought 6 beach towels at $$x$ each. Express the amount of money she had left in terms of x. If $x = 17$, how much money did she have left?

Using Variables with Tape Diagrams

Example 5:

Max had brownies. He ate 4 brownies and shared the remaining brownies among his 6 friends equally? How many brownies did each friend receive? Express your answer in terms of . If , how many brownies did each friend receive?

Grade 7 – Module 3 – Lesson 7

Solve the problem first with a tape diagram, then an equation.

The ages of three sisters are consecutive integers. The sum of their ages is 45. Find their ages.

Grade 7 – Module 3 – Lesson 8

Solve the problem first with a tape diagram, then an equation.

Julia, Keller, and Israel are volunteer firefighters. On Saturday the volunteer fire department held its annual coin drop fundraiser at a streetlight. After one hour, Keller had collected $42.50 more than Julia, and Israel had collected $15 less than Keller. Altogether, the three firefighters collected $125.95. How much did each person collect.

Grade 7 – Module 4 – Lesson 2

Solve the problem first with a tape diagram, then an equation.

In Ty's art lass, 12% of the Flag Day art projects received a perfect score. There were 25 art projects turned in by Ty's class. How many of the art projects earned a perfect score?

Grade 7 – Module 6 – Lesson 1

Solve the problem first with a tape diagram, then an equation.

The measures of two supplementary angles are in the ratio of 2:3. Find the measures of the two angles.

Grade 7 – Module 6 – Lesson 1

Solve the problem first with a tape diagram, then an equation.

In a pair of complementary angles, the measurement of the larger angle is three times that of the smaller angle. Find the measures of the two angles.

Algebra I – Module 1 – Lesson 25

Solve the problem first with a tape diagram, then an equation.

In a school choir, one-half of the members were girls. At the end of the year, 3 boys left the choir, and the ratio of boys to girls became 3:4. How many boys remained in the choir.

Algebra I – Module 1 – Lesson 25

All the printing presses at a print shop were scheduled to make copies of a novel and a cookbook. They were to print the same number of copies of each book, but the novel had twice as many pages as the cookbook. All of the printing presses worked for the first day on the larger book, turning out novels. Then, on day two, the presses were split into two equally sized groups. The first group continued printing copies of the novel and finished printing all the copies by the evening of the second day. The second group worked on the cookbook but did not finish by evening. One printing press, working for two additional full days, finished printing the remaining copies of the cookbooks. If all printing presses printed pages (for both the novel and cookbook) at the same constant rate, how many printing presses are there at the print shop?

Key Points

- When building proficiency in tape diagraming skills start with simple accessible situations and add complexities one at a time.
- Develop habits of mind in students to reflect on the size of bars relative to one another.
- Part-whole models are more helpful in modeling situations where:_____
- Compare to models are more helpful in modeling situations where: _____

Session Objectives

- Experience how proficiency in the tape diagram method can be developed in students and colleagues.
- Experience how to support understanding of various types of word problems as outlined in the Progressions document.
- Mathematical Practice 5: *Use appropriate tools strategically* and demonstrate this knowledge by using a tape diagram to solve problems.

Feedback

- Now that you've experienced these two days, comment on what you learned each day.
- Comment on the order of the sessions (Day 1: Major Works, Day 2: Tape Diagrams).
- We welcome any other comments.