# Problem of the Month: *Circular Reasoning*

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

**Overview:**
In the Problem of the Month *Circular Reasoning,* students use geometric reasoning to solve problems involving circles and the number π. The mathematical topics that underlie this POM are the attributes of polygons, circles, the irrational number π, spatial visualization, and angle measurement.

In the first level of the POM, students are presented with the task of examining the relationship between the measure of the diameter of a circle and its circumference. Their task involves determining the number of times bigger the circumference is than the diameter (approximation of π). Level B requires students to continue to

investigate the number π by investigating the relationship between the measurements of a tennis ball can. They find the height of the can and the circumference of the base of the can. They reason why the two measurements are close to the same size and search for a pattern to determine a rationale. In level C, students investigate different size pizzas made in the form of a circle. The pizzas are divided into fractional slices shaped like a sector of a circle. The students investigate the relationships between the sizes of the pizzas, including the dimensions of the slices (radii and degrees), the costs, and whether the costs are proportional to their sizes. In level D, the students investigate the rings that support wooden barrels. In this problem, the rings are made a foot longer than they were supposed to be. Since the barrels come in a range of sizes from very large to very small, the foot long error must be explored. The goal is to determine the relationship between error in rings to the original size and how the extra length affects a redesigned barrel. In level E, students investigate Archimedes' approach to calculating an approximation for π. Students are asked to duplicate the drawings and calculations of Archimedes and determine the accuracy of his approximation. They are also asked to justify the formula for the area of a circle.

**Mathematical Concepts:**
Geometry and measurement are important real-world experiences. From the most complex structures created by designers, architects, and construction workers to arranging the furniture in a room, geometry and measurement are essential. In this POM, students explore various aspects of circular geometry. This includes understanding the attributes of circles. In particular, students study regular polygons, diameters, circumference, and angles. The attributes studied include the diagonals of polygons as well as angles and their measurements. Students explore the irrational number π. In level E, students use an historical approach to calculate an approximation of π. In addition to the geometric and measurement aspects of this POM, the students are seeking to find patterns, develop functional relationships, make generalizations, and justify their conclusions. The mathematics involves higher-level cognitive skills.

# $\pi$ Problem of the Month
# Circular Reasoning $\pi$

**Level A**

Janet and Lydia want to learn more about circles. They decide to measure different size circles that they can find. They measure the circles in two ways. One way is across a centerline of the circle. The centerline is called the diameter. Another way is around the outside of the circle. The distance around the outside of a circle is called the circumference.

Measure various size circles using either physical objects or the included page of circles. Measure both the circumference and diameter of each circle with a tape measure (paper or cloth tape). Create a table that compares the length of the diameter with the circumference of the circle. Measure at least five different size circles.

| Diameter | Circumference |
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Compare the numbers between the two columns. What patterns do you see?

Explain any relationship you see between the length of the diameter and the circumference.

**Level B**

Measure a tennis ball can.  First measure the height of the can, from the top of the can to the bottom.  Record the measurement.

Next, measure the circumference around the can.  Be accurate with your measurements.

Compare the measurements.

Explain the relationship between the two measurements.

How does the number of balls in the can relate to the measurements and their relationship?

What other parts of the can or tennis balls can you measure and compare?
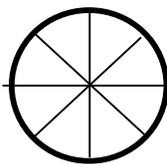
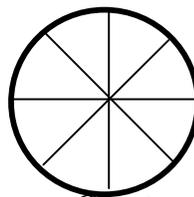Describe the relationship of those measurements?  How are they related?

**Level C**

You work for a pizza parlor. You currently have three sizes of pizza: regular (area 25 square inches), large (area 36 square inches) and giant (area 64 square inches).
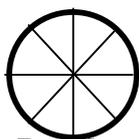


**Regular
25 sq. in.**

**Large
36 sq. in.**

**Giant
64 sq. in.**

The storeowner has asked you to determine some information about the amount of pizza that is made.

How much bigger is the giant pizza than the large pizza in terms of area? How much bigger is the large pizza than the regular pizza in terms of area?

The owner sells a regular cheese pizza for $8.25. He wants to know what to charge for the large and giant sized pizzas. He wants to cover the cost of the ingredients and make a proportional amount of profit on the other two pizzas. What should he charge for a large and giant cheese pizza?

The owner sells pizza by the slice. He has been selling a large pizza by the slice for $1.35. Each pizza is cut into eight equal slices. The owner wants to know whether he has priced it right to make more money selling by the slice than the whole pizza. Explain to him, using mathematics, how his prices for the slices and the whole pizza compare.

Now the owner wants to cut each pizza into nine slices, instead of eight. How much smaller will the slices be? He is interested in comparing both the width (angle in numbers of degrees) and the area of each slice.

The owner wants to sell the nine slices at the same price as he did when the pizza was cut into eight slices. He wants to know how much more he will make cutting the pizzas into smaller slices. He wants to compare the prices of selling the nine slices individually with the price of selling an entire large pizza. He asks you to calculate and explain the difference in prices and amounts.

The owner also thinks he can save more money if he sells slices from regular sized pizzas instead of large sized pizzas. You cut both the regular and large pizzas into nine equal slices. How much larger is a single slice from a large pizza than that of a slice from a regular pizza? Explain your answer in terms of degrees of the angle and area of the slice.

**Level D**

You are a quality controller for a barrel manufacturing company. Your company makes barrels of all different sizes. The company makes very large storage vats for vineyards. They even make very small barrels as earrings that are sold in the gift shop next to the factory. Without exception, the company makes every barrel the same way. They use wood supported with metal rings that go around the barrels. A computer controls the ring-making machine. The computer malfunctioned and made every ring, from the smallest for the earrings to the largest for the wine vats, larger than normal by exactly one foot. In other words, every ring your factory has turned out recently has a circumference that is twelve inches longer than the right size. Your boss has asked you to investigate this matter.



How does the altered rings compare to the original rings? How much bigger do the altered rings look in comparison to the rings on the barrels that they were designed to fit? How will the altered rings compare in size to the rings on the small barrels, like the earrings? How will the altered rings compare to the rings on the large barrels, like the wine vats?

What should the company do with all those altered rings? It would be very costly to dispose of them. Your boss has asked you to tell him what size barrels need to be made to use those rings. Are the barrels usable? What size would you have to make the new barrels in relationship to the original size barrels?

What conclusion can you draw from your findings? Can you explain your findings using mathematics? Why are the results what they are?

**Level E**

Archimedes is considered one of the greatest mathematicians of all time. He lived in the Greek city of Syracuse, Sicily between 287 BC and 212 BC and is known for his work in many areas of mathematics and science.

Archimedes mathematical work includes inventing methods of calculating the area and volume of geometric objects, such as the circles, parabolas, cylinders, cones, spheres, hyperbolas and ellipses. He also spent time investigating a rational number approximation of π. The method he used to approximate π involved examining the size of many-sided polygons. Archimedes, as the story is told, drew a large circle on the floor of his apartment. He measured the diameter of the circle and called that distance one unit in length. Next he began to inscribe and circumscribe different size polygons, tangent to the circle. Archimedes would measure the perimeters of the polygons and average the two to approximate π. As the numbers of sides of the polygons were increased, the shaped of the polygon looked more like the circle. The polygons got closer and closer to being the size of the circle and perimeters became ever closer to the value of π. Using that method, he was able to calculate the approximation of π, correct to a value between 3 1/7 and 3 10/71.

How precise was his approximation of π? Give your answer in terms of the number of correct decimal places.

Estimate how many sides his final polygon needed to be to get a number that accurate? Explain your estimate.

Illustrate the method Archimedes used by drawing an interior regular polygon inscribed in a circle and an exterior regular polygon circumscribed about the same circle. Determine the perimeter of both polygons and show how π might be approximated.

Find the area of both polygons and compare the areas with that of the circle. Explain how the area formula is derived from the area formula of the polygon.

# $\pi$ Problem of the Month Circular Reasoning $\pi$

## Primary Version Level A

**Materials:** A picture of a circle, circular objects (i.e. counters, coins), the $\pi$-day picture, paper and pencil.

**Discussion on the rug:** (Teacher holds up a picture of a circle) "What is the name of this shape?" (Students respond to the question). (Teacher hands out circular objects) "These objects are shaped like a circle. Tell us about the shapes?" (Students explore the objects and describe the circles). (Teacher holds up a circle) "A circle is a special shape. Long ago people discovered the circle and tried to measure it. They found a special number called pi. Some people who like math celebrate circles on a special day, March 14. It is called pi day. Here is a picture of people having a picnic on pi day." (Teacher hands-out the picture of the $\pi$-day picnic).

**In small groups:** (Each student has the picture of the $\pi$-day picnic). "Look at the picture of the picnic. Find all the things in the picture that are shaped like a circle. Circle each shape you find." (Students answer the following questions).
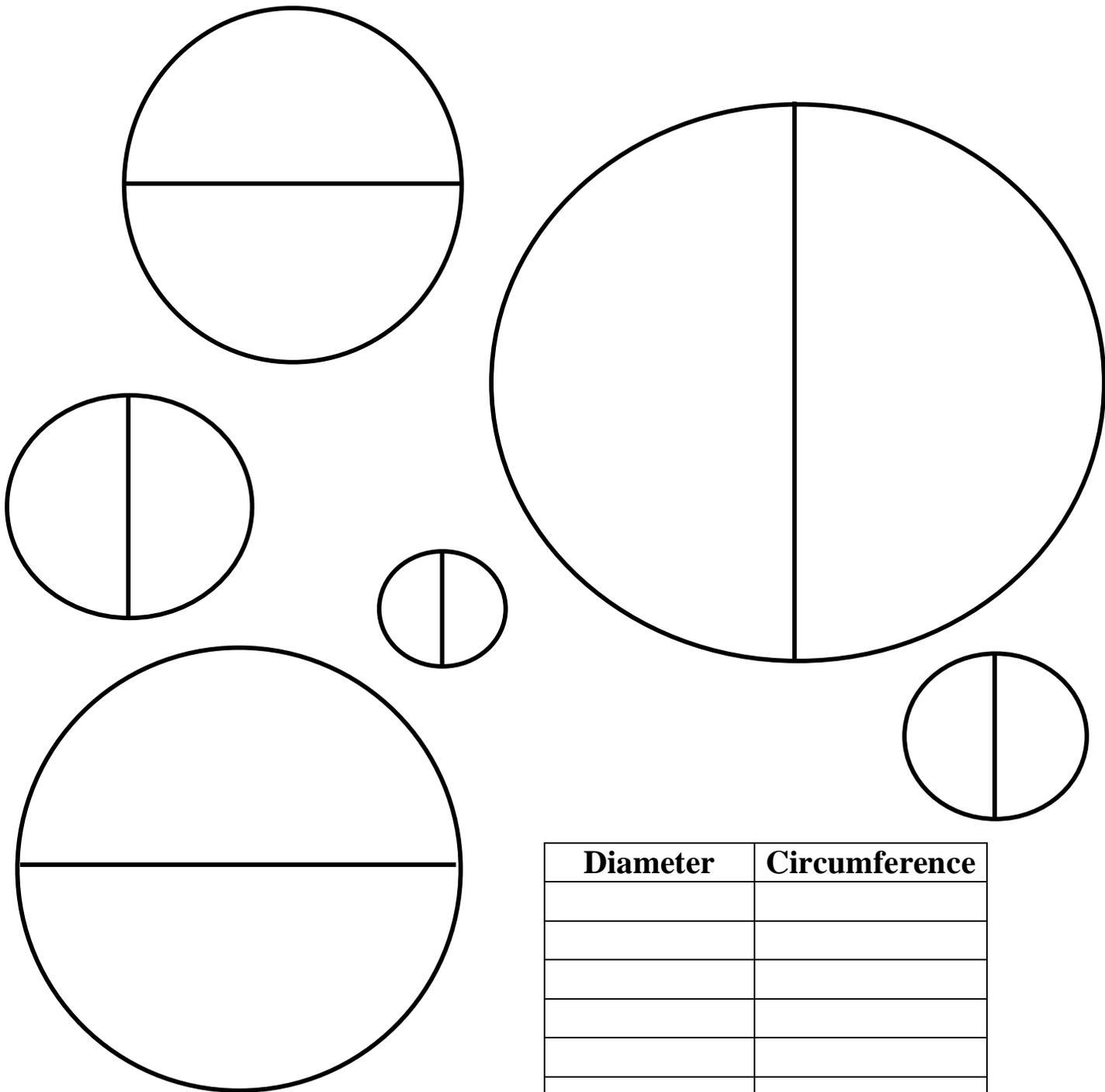How many things did you find that are shaped like a circle?
Name the things in the picture that are shaped like a circle.
(At the end of the investigation, have students either discuss or dictate a response to this summary question below.)
Explain how you decided which ones to circle.

π day picnic

| Diameter | Circumference |
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| Problem of the Month |
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| ***Circular Reasoning*** |
| Task Description – Level A |

This task challenges a student to explore various aspects of circular geometry. Students are asked to examine the relationship between the measure of the diameter of a circle and its circumference. Their task involves determining the number of times bigger the circumference is than the diameter (approximation of $\pi$).

| Common Core State Standards Math - Content Standards |
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**Geometry**
**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.)**
K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*

K.G.2 Correctly name shapes regardless of their orientations or overall size.

**Measurement and Data**
**Describe and compare measurable attributes.**
K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe measurable attributes of a single object.

**Measure and estimate lengths in standard units.**
2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

**Operations and Algebraic Thinking**
**Generate and analyze patterns.**
4.OA.5 Generate a number or shape pattern that follows a given rule. ...

**Expressions and Equations**
**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

**Functions**
**Define, evaluate, and compare functions.**
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. ...

**Use functions to model relationships between quantities.**
8.F.4 Construct a function to model a linear relationship between two quantities. ...

**High School – Functions - Building Functions**
**Build a function that models a relationship between two quantities.**
F-BF.1 Write a function that describes a relationship between two quantities.
 a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

| Common Core State Standards Math – Standards of Mathematical Practice |
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**MP.2 Reason abstractly and quantitatively**.
Mathematically proficient students make sense of quantities and their relationships in problem

situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**MP.4 Model with mathematics.**
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

| Problem of the Month |
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| *Circular Reasoning* |
| Task Description – Level B |

This task challenges a student to explore various aspects of circular geometry. A student is required to continue to investigate the number π by investigating the relationship between the measurements of a tennis ball can, such as the height of the can and the circumference of the base of the can. They are to reason why the two measurements are close to the same size and to search for a pattern to determine a rationale for this relationship. Additionally, a student is asked to explore and describe the relationship between the number of balls in a can and the height of a can.

| Common Core State Standards Math - Content Standards |
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**Geometry**
**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**
K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*

K.G.2 Correctly name shapes regardless of their orientations or overall size.

**Measurement and Data**
**Describe and compare measurable attributes.**
K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe measurable attributes of a single object.

**Measure and estimate lengths in standard units.**
2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks,  meter sticks, and measuring tapes.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

**Operations and Algebraic Thinking**
**Generate and analyze patterns.**
4.OA.5  Generate a number or shape pattern that follows a given rule. ...

**Expressions and Equations**
**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

**Functions**
**Define, evaluate, and compare functions.**
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. ...

**Use functions to model relationships between quantities.**
8.F.4 Construct a function to model a linear relationship between two quantities. ...

**High School – Functions - Building Functions**
**Build a function that models a relationship between two quantities.**
F-BF.1 Write a function that describes a relationship between two quantities.
  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

| Common Core State Standards Math – Standards of Mathematical Practice |
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**MP.2 Reason abstractly and quantitatively**.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**MP.4 Model with mathematics.**
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

| Problem of the Month |
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| ***Circular Reasoning*** |
| Task Description – Level C |
| This task challenges a student to explore various aspects of circular geometry.  A student is asked to investigate different size pizzas made in the form of a circle. The pizzas are divided into fractional slices shaped like a sector of a circle. The students investigate the relationships between the sizes of the pizzas, including the dimensions of the slices (radii and degrees), the costs, and whether the costs are proportional to their sizes. |
| Common Core State Standards Math - Content Standards |

**Operations and Algebraic Thinking**
**Represent and solve problems involving addition and subtraction.**
2.OA.1  Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, …

**Ratio and Proportional Relationships**
**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.3
   c.  Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.1  Recognize and represent proportional relationships between quantities.

7.RP.3  Use proportional relationships to solve multistep ratio and percent problems.  …

**Geometry**
**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
7.G.4  Know the formulas for the area and circumference of a circle and use them to solve problems…

7.G.5  Use facts about …angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

| Common Core State Standards Math – Standards of Mathematical Practice |
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**MP.4 Model with mathematics.**
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are

comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

| Problem of the Month: |
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| ***Circular Reasoning*** |
| Task Description – Level D |
| This task challenges a student to explore various aspects of circular geometry.  A student is asked to investigate the rings that support wooden barrels. In this problem, the rings are made a foot longer than they were supposed to be. Since the barrels come in a range of sizes from very large to very small, the foot long error must be explored. The goal is to determine the relationship between error in rings to the original size and how the extra length affects a redesigned barrel. |

## Common Core State Standards Math - Content Standards

**Geometry**
**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
7.G.4  Know the formulas for the area and circumference of a circle and use them to solve problems…

7.G.6  Solve real-world and mathematical problems involving area, volume and surface area of two- and three- dimensional objects…

**Ratio and Proportional Relationships**
**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.2  Recognize and represent proportional relationships between quantities.

7.RP.3  Use proportional relationships to solve multistep ratio and percent problems.  …

**Expressions and Equations**
**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.4  Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
   a.  Solve word problems leading to equations of the form *px+q=r* and *p(x+q)=r*, where *p, q, and r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

## Common Core State Standards Math – Standards of Mathematical Practice

**MP.2 Reason abstractly and quantitatively**.
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**MP.4 Model with mathematics.**
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of

interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

| Problem of the Month |
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| ***Circular Reasoning*** |
| Task Description – Level E |
| This task challenges a student to explore various aspects of circular geometry.  A student is asked to investigate Archimedes' approach to calculating an approximation for π.  Students are asked to duplicate the drawings and calculations of Archimedes and determine the accuracy of his approximation. They are also asked to justify the formula for the area of a circle. |
| Common Core State Standards Math - Content Standards |

**Geometry**
**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
7.G.4  Know the formulas for the area and circumference of a circle and use them to solve problems...

7.G.6  Solve real-world and mathematical problems involving area, volume and surface area of two- and three- dimensional objects...

**High School – Geometry - Congruence**
**Make geometric constructions.**
G-CO.12  Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G-CO.13  Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

**High School – Geometry - Similarity, Right Triangles, and Trigonometry**
**Apply trigonometry to general triangles.**
G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

| Common Core State Standards Math – Standards of Mathematical Practice |
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**MP.4 Model with mathematics.**
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situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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| Problem of the Month |
| *Circular Reasoning* |
| Task Description – Primary Level |
| This task challenges a student to explore various aspects of circular geometry. This level asks students to describe and identify circular shapes in real world objects such as coins, counters, containers, etc. Then students are asked to ring and identify all shapes in a given picture which are shaped like a circle. |
| Common Core State Standards Math - Content Standards |
| **Geometry**<br>**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**<br>K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*<br><br>K.G.2 Correctly name shapes regardless of their orientations or overall size. |
| Common Core State Standards Math – Standards of Mathematical Practice |
| **MP.2 Reason abstractly and quantitatively**.<br>Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.<br><br>**MP.4 Model with mathematics.**<br>Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. |