Problem of the Month: *On Balance*

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:
In the Problem of the Month *On Balance*, students are engaged in tasks and puzzles that involve equality, inequalities, equations, and simultaneous constraints. The mathematical topics that underlie this POM are measurement, number sentences, equality, inequality, variables, inverse operations, and simultaneous systems.

In the first level of the POM, students are presented with a puzzle involving the weight of three apples and a balance scale. The task is to find which apple is lighter

in only one weighing. They need to use logic and their knowledge of equality and inequality. In level B, students are presented with five apples of which one is bad. They know the bad apple is a different weight than the other four, but are not sure whether the bad one is heavier or lighter. Students need to use the balance scale to determine the bad apple in the least amount of weighs. In level C, students are presented with a set of balance scales showing how different kinds of fruit weigh in relationship to one another. The students use their knowledge of equality and equations to determine how the strawberries compare to the limes in weight. In level D, students are given a similar situation to the one they had in level B, but this time they have nine apples to weigh in order to determine the bad apple. The goal is to find the bad apple in the least number of weighs. The students are asked to make a generalization based on the number of apples to consider. In the final level E, students are asked to consider five simultaneous equations in the form of fruit on a balance scale. The task is to find the weights of 6 different fruits given that the strawberries weigh 3 oz.

Mathematical Concepts:
Algebra is the cornerstone of secondary mathematics. Algebraic thinking is taught in primary grades with the foundations of algebra taught usually by the end of middle school. Even though the term algebraic thinking is routinely used, it can not be simply defined. The underpinnings of algebra involve abstractions and language. There are several resources that define the most important concepts in algebra. One resource is *Fostering Algebraic Thinking.*
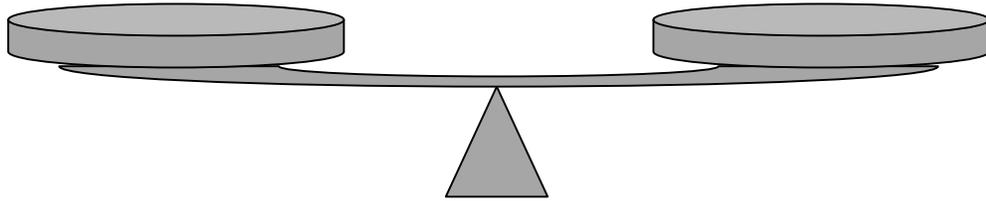
On Balance

Level A

Debbie works at an apple orchard. She knows every so often there is a bad or rotten apple in the baskets of apples she picks. She knows that bad apples weigh less than good ones. There are a lot of apples in her basket so she wants to spend as little time as possible for checking apples. She thinks she knows a fast way to check.
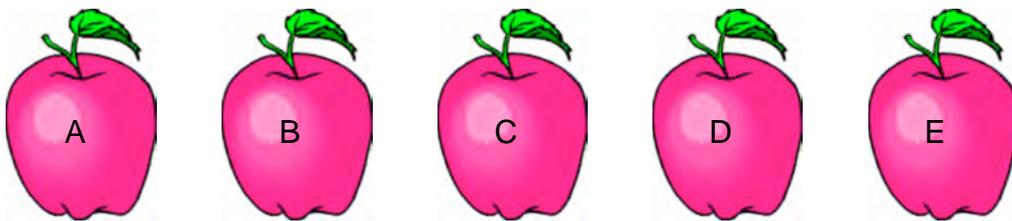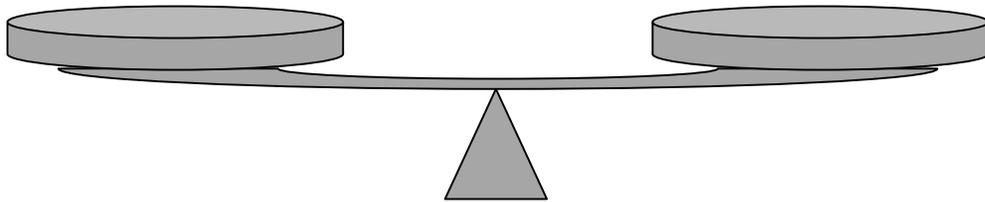
She has three apples and one is bad. How can she weigh apples just one time and still find out which one is bad.

Making only one weighing, can you determine a way to know for sure which apple is bad? Explain.
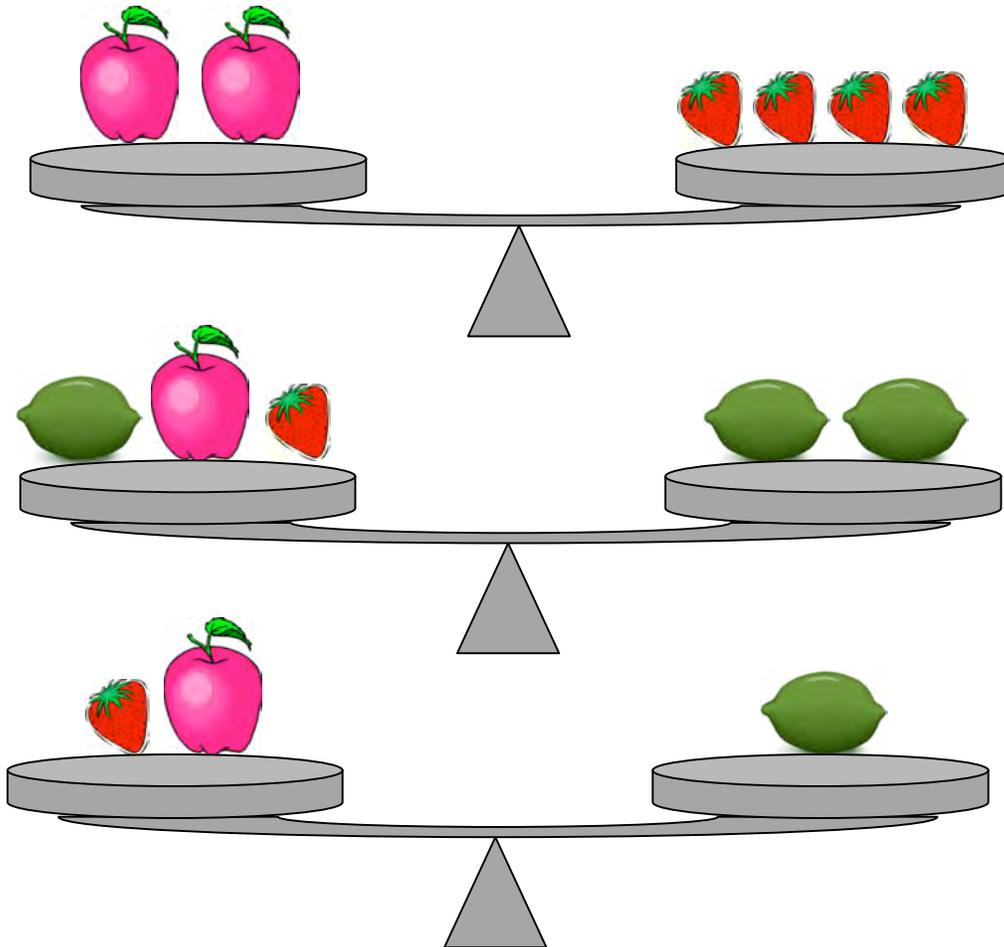
Level B

Debbie has a lot more apples to check. Now she has five apples. Four apples are good, all weighing the same. But the bad one either weighs more or weighs less than the others. List the process steps and decision you need to make in order to determine which apple is bad, and whether it is heavier or lighter than the good ones.

You know one of the apples is either heavier or lighter than all the others, what are the minimum weighs needed to determine which apple is bad, and whether it is heavier or lighter than the others. Show and justify your solution.
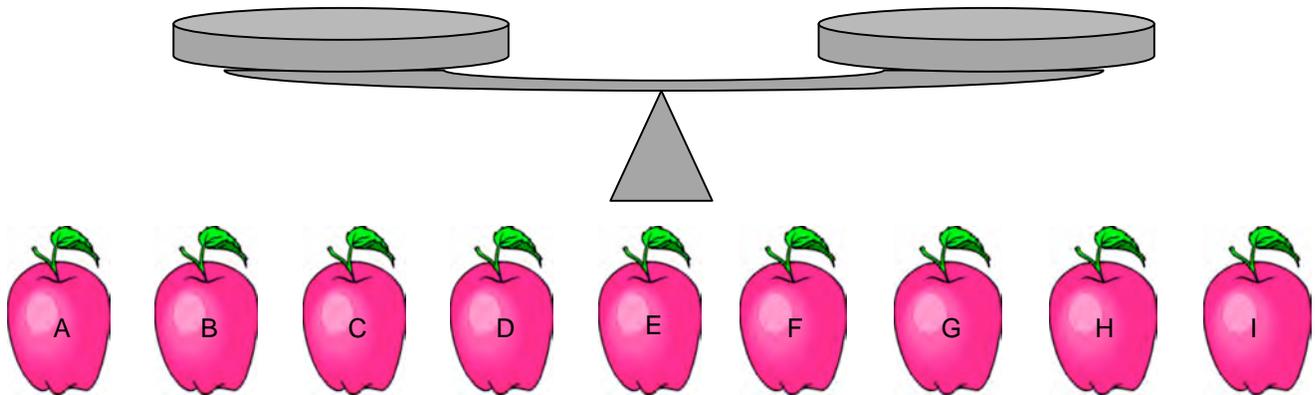
Level C

Each type of fruit has a unique weight that is consistent between scales. Use the scales to determine the relationship between the different types of fruit.

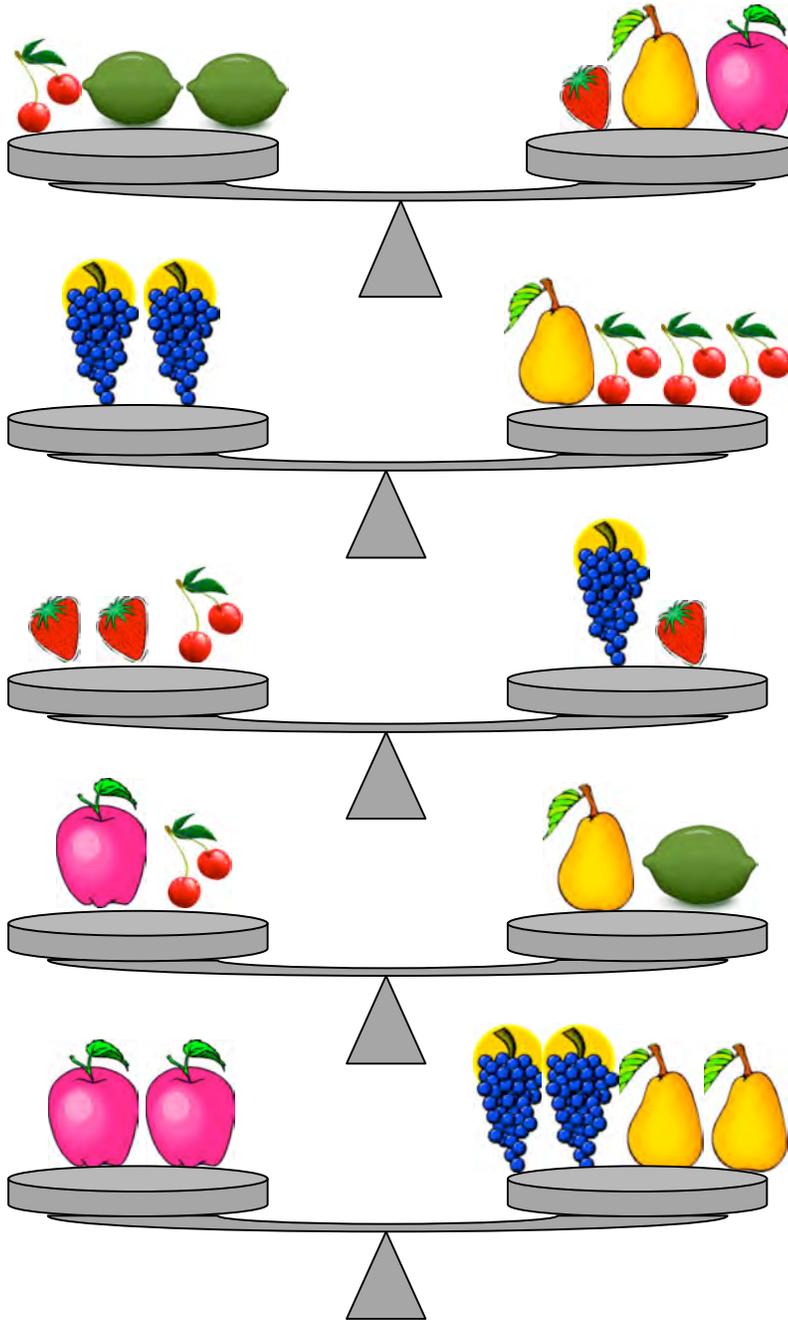How do the weights of strawberries compare with limes?

Level D

Suppose you were presented with nine apples (A-I). Eight apples are the same weight and the ninth, either weighs more or weighs less than the others. List the process steps and decision you need to make in order to determine which apple is different, and whether it is heavier or lighter than the others.

You are given any number of apples between 3 and 15 and you know that one of the apples is either heavier or lighter than the others, which weigh the same. For each set of apples, what are the minimum number times you need to weigh the items to insure that you find the one that weighs different than the others? Show the relationship between the number of apples and the minimum weighing needed.

Level E

Each type of fruit has a unique weight that is consistent between scales. Use the five scales to determine the relationship between the different types of fruits. How do the weights of each fruit compare with each other? Which weighs the most? Which weighs the least? Suppose a strawberry weighs 3 oz., how many of one kind of fruit equal to another kind of fruit? Explain.

Problem of the Month
On Balance

Primary Version Level A

Materials: A scale and three objects of different weights (maybe an apple, pear and orange), paper, and pencil.

Discussion on the rug: (Teacher shows a balance scale) "What do we use this object for?" (Students volunteer ideas. If students don't know, then the teacher puts two different objects on the scale and asks the question). "What do you think it tells us now?" (Students respond with ideas) (Teacher asks) "Okay, which object is heavier?" (Students respond) "How do we know?" (Teacher asks) "Which one weighs the most? Which one weighs the least?

In small groups: (Each student pair has a scale and three different objects – maybe a fruit, a piece of paper and a pencil. The teacher explains that they can use the scale to find the answer to these questions.)
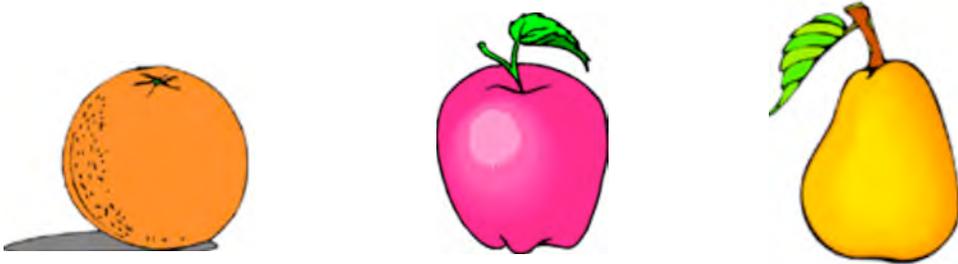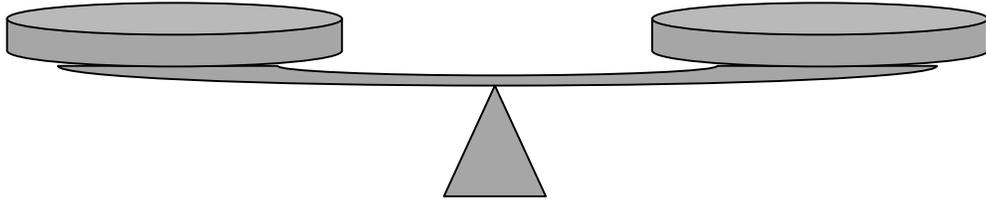Which object weighs the most?
Which object weighs the least?
What do we know about the third object (fruit)?
Put the objects in order from heaviest to lightest?
 (At the end of the investigation have students either discuss or dictate a response to this summary question.)
How did you figure out which object (fruit) weighed the most? Tell me how you figured it out and how you know for sure.

Problem of the Month
On Balance
Task Description – Level A
This task presents a student with a puzzle involving the weight of three apples and a balance scale. A student is challenged to find which apple is the lighter after only one weighing. A student will need to use logic and their knowledge of equality and inequality to help them.
Common Core State Standards Math - Content Standards

Measurement and Data
Describe and compare measurable attributes.
K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP.7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression x^2 + 9x + 14, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single

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objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $\mathbf{x}$ and $\mathbf{y}$.

MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x–1)(x+1),(x–1)(x2+x+1),and(x–1)(x3 +x2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
On Balance
Task Description – Level B
This task challenges a student to find which apple out of five apples is the bad one. A student realizes that the bad apple may be heavier or lighter but they aren't sure. A student will need to use the balance scale to determine the bad apple in the least number of weighs on the scale.
Common Core State Standards Math - Content Standards

Statistics and Probability
 Investigate chance processes and develop, use, and evaluate probability models.
 7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
    a.   Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
    b.   Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Common Core State Standards Math – Standards of Mathematical Practice

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MP. 7 Look for and make use of structure.
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and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $\mathbf{x}$ and $\mathbf{y}$.

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Problem of the Month
On Balance
Task Description – Level C
When presented with a set of balance scales showing different kinds of fruit and their weights in relation to each other.   A student is challenged to use this information and their understanding of quality and equations to determine how the strawberries compare to the limes in weight.
Common Core State Standards Math - Content Standards

Measurement and Data
Describe and compare measurable attributes.
K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Common Core State Standards Math – Standards of Mathematical Practice

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objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $\mathbf{x}$ and $\mathbf{y}$.

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Problem of the Month:
On Balance
Task Description – Level D
This task challenges a student to determine which apple out of nine apples is the bad one (i.e., which apple has a different weight than the other eight apples) in the least amount of weighs on the scales. This problem is similar to Level B. A student is asked to make a generalization based on the number of apples to consider.
Common Core State Standards Math - Content Standards

Measurement and Data
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K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.
 	
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Problem of the Month
On Balance
Task Description – Level E
This task challenges a student to consider five simultaneous equations in the form of fruit on a balance scale. A student is to find the weight of the six different fruits given that the strawberries weigh 3 oz.
Common Core State Standards Math - Content Standards
Expressions and Equations **Analyze and solve linear equations and pairs of simultaneous linear equations.** 8.EE.8 Analyze and solve pairs of simultaneous linear equations.   c.   Solve real-world and mathematical problems leading to two linear equations in two variables. **High School - Algebra - Creating Equations** **Create equations that describe numbers or relationships.** A-CED.2 Create equations in two or more variables to represent relationships between quantities…
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.6 Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. **MP. 7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression x^2 + 9x + 14, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview

and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $\mathbf{x}$ and $\mathbf{y}$.

MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1), (x-1)(x^2+x+1),$ and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
On Balance
Task Description – Primary Level
This task challenges a student to work with a scale and three different fruits with different weights. The teacher holds a discussion about a balance scale – asking students what they know about it and specifically when two different objects are placed on the scale. The teacher asks which object is heavier and how does a student know. Then, in small groups, students have 3 objects which they weigh and decide which one weighs the most, the least, and orders them from the heaviest to the lightest. At the end of the task, the whole class has a discussion on how the frames are alike and different. At the end of the investigation a student will dictate a response to the question, "How do you figure out which object weighs the most?"
Common Core State Standards Math - Content Standards
Measurement and Data   **Describe and compare measurable attributes.**   K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.   K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others.   Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.   **MP.6 Attend to precision.**   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.   **MP. 7 Look for and make use of structure.**   Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students

will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3 +x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.